

## FRACTALS – A SMART FINANCIAL TOOL TO ASSESS BUSINESS MANAGEMENT DECISIONS

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### ABSTRACT

*The research aims to identify the existence and connections between financial decision, financial performance and fractals patterns. The investigation starts to examine if there is the possibility to verify a repeating pattern for future events to be foreseen. Mathematics is a logical base of our entire life, and fractals are of it. A fractal appear as a geometric object that repeats itself in the same structure on different scales and/or timings, showing reduplicate patterns. Fractals are everywhere around us. Fractal patterns can be easily linked with [financial markets](#) or any economic events. Price movement in the market are very complex and appears to be randomly. The financial economists believed that this complexity is built up from self-similar patterns of trader behavior. That means that the whole structure is not random but follows a model that can be measured – fractals. A multidimensional analysis of financial data provides a clear picture of decision makers at company level or at macroeconomic level. Economic data are very useful and they can be classified according to different decision maker criteria and taking into consideration accordingly. However for a better performance of economic and financial forecasts, predictions and the impact of any decisions throughout the economy, we argue that fractals pattern are more than helpful be taken into account.*

**KEYWORDS:** *fractals; financial decisions; intelligent management decisions; smart forecasting.*

### 1. INTRODUCTION

This research purposes to study if there is the possibility to verify a repeating pattern for future events to be foreseen. In particular, based on the Mandelbrot's studies, [1] taken up and applied by the economist Taleb [2], it has been possible to hypothesize the occurrence

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of improbable events, which may have a devastating impact. The Authors focused on the comparison between the original fractal approach and the Gaussian one, based on the so-called *Gaussian bell*. The latter excludes the possibility of considering some events simply because they are unlikely, regardless of the effects they might produce.

A fractal is a geometric object that repeats itself in the same structure on different scales and/or timings, showing repeating patterns. It does not change its appearance even if it is viewed with a magnifying glass. Objects with such behavior could appear as artificial constructions, although they are frequent in nature such as: the arrangement of the branches of a tree, the shape of a cauliflower, the surface of the clouds, the path of a river, the structure of the galaxies, the shape of lightning (figure no.1).



Figure 1. Examples of fractals shapes

Furthermore, since its Mandelbrot's discover, fractals are becoming a powerful new mathematical language, thanks to which it is possible to describe natural phenomena and solve the real natural problems that had once been set aside. The Mandelbrot's crucial aim it was to find a comprehensive way to explain the chaos. This modern mathematics makes use of the information technology (Mandelbrot used to work at IBM for decades). To understand the importance of fractal pattern, it is necessary to take a step back in time. Galileo Galilei, one of the greatest scientists of all time, believed that mathematics was an indispensable discipline for interpreting natural phenomena and for representing the forms of nature. However, our daily experience leads us to believe that the most familiar geometric figures (lines, circles, regular polygons) are exception in the nature. This is precisely the Mandelbrot's objection [3], which introduced the fractals in 1975 as new geometric figures more efficient to represent the complexity of nature. The term *fractal*, which he invented, it comes from the Latin word *fractus* (broken, fractionated). Fractals are geometric figures that may seem irregular because they cannot be traced back to the classical figures of Euclidean geometry. They are in fact strange figures, very jagged, grainy, sometimes ramified and intricate, with tentacles or protuberances, just like most of the figures in nature.

A fractal is a geometric object with internal homothetic, which it repeats his shape in its same form on different scales. Enlarging any part, it is possible to obtain a similar figure to the original. It is therefore called *fractal geometry*, the Non-Euclidean geometry that studies these structures, recurrent for example in the engineering design of networks, in Brownian motion and in galaxies. This feature is also known as *self-similarity*. Benoit Mandelbrot invented the term *fractal* in 1975 and described like *imagine of a figure, a leaf for example, which reproduces itself to infinity, always the same shape but always*

*smaller*. In this way the fractal can be used in the description of real natural events or objects. Moreover, the fundamental characteristic of the fractal figures is *self-similarity*: if the details are observed on different scales, it is always possible to notice a certain resemblance to the original fractal. Fractal geometry comes to identify these configurations, to analyze and manipulate them and can be used, not only as a tool for analysis and synthesis, but also as a forecast. The rules of the fractals are precise, so the result is predictable. This contrasts with traditional science which instead consider the irregular aspects of nature and non-similar events such as chaos theory [4]. The chaos theory is considered like a drop of water that expands into the sea, or cardiac fibrillations, or even computer errors and price fluctuations.

The Mandelbrot's set (figure no.2) is one of the most popular fractals, also known outside the mathematical field due to the suggestive multicolored images that have been disclosed. It is the set of complex numbers  $c$  that shows a limited sequence defined by:

$$\begin{cases} z_0 = 0 \\ z_{n+1} = z_n^2 + c \end{cases}$$

Despite the simplicity of the definition, the function has a complex shape whose contour is a fractal. It possible to view and draw it only by using a computer.

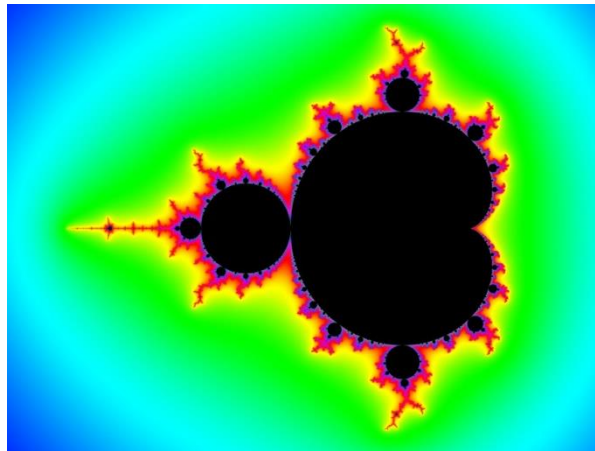


Figure 2. The Mandelbrot's Set

## 2. RESEARCH METHODOLOGY

This research is proposed to be a qualitative exploratory research, because the aim of the paper is to deepen the already advanced researches made by [1] and Taleb [2], focusing on the theory of finance and linking it to the business management. The research started from the obvious existence of fractals in everything that surrounds us and wishes to show and confirm that patterns exist in economic life and may help, even may become an important tool for financial decision makers. Fractals can let financial experts to be aware or prepared for periods of cyclical recession.

### 3. LITERATURE REVIEW

#### 3.1. Different approach: Fractal versus Gaussian bell

Now become understandable that fractals have always there, part of our day-to-day life, just waited to be discovered. At the beginning of the twentieth century, some mathematicians had created very strange curves and figures that subverted the rules of classical geometry violating the characteristics of harmony considered natural for objects in the scientific field. This is a case of *Koch's snowflake* (a line which link all corners – figure no.3); *Peano-Hilbert curves* (a line that unfold a labyrinth which it covers a square – figure no.4); pitted figures, like the *Cantor's set* (figure no.5); or the Sierpinski's carpet (Figure no. 6). At that time, these structures were considered as monsters to be relegated in a sort of museum of horrors or to be exhibited only in an equestrian circus.

Thanks to Mandelbrot, the *mathematical monsters*, previously set aside, were finally considered, assuming a new role of ancestors of modern fractal figures. Fractals were therefore born by recovering pre-existing separate pieces, but conceived in limited and distinct contexts. The mathematicians were surprised and pleased to discover that their pathological figures had become the key to the complexity chased for so long. In the last twenty years, fractal models have been investigated, acquiring the role of the key structure in financial modeling. Mathematic in all sectors is constantly growing: from natural sciences to economics and social sciences, from physiology to technology, advanced logistics and their field of application.

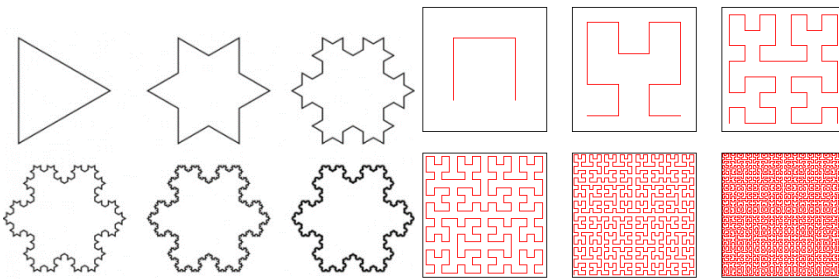


Figure 3. Koch's Snowflake

Figure 4. Peano-Hilbert curves

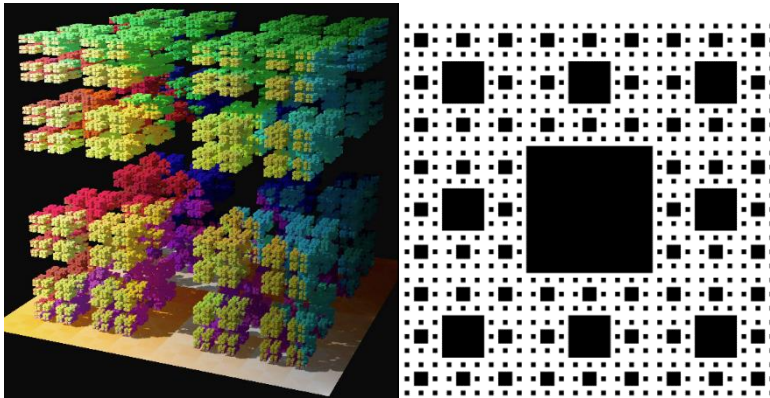


Figure 5. 3D Cantor's set

Figure 6. Sierpinski's carpet

The fractals theory is also closely linked to the chaos theory. Thanks to the captivating graphic representation that can be obtained with the help of a computer, fractals have also acquired a space in the art world. Therefore, fractal painting and music were born. Beyond their apparently very complex figures, secret of extreme simplicity is hidden. This is perhaps the most surprising aspect of the whole fractal theory.

Karl Friedrich Gauss' mathematic formulas and everything that is behind the mathematician is known to a few of us, however its general meaning and usefulness is known to many. *Gaussian curve* is a curve with a classic bell shape that has a maximum around the average of the measured values and can be more or less narrow depending on the dispersion of the values around the average. The dispersion is measured with the standard deviation: practically one of the properties of the Gaussian is that 68% of the measurements differs from the average less than the standard deviation and that 95% less than two standard deviations. Therefore, the greater the standard deviation, the more the Gaussian it is *open* and there is the possibility that the average (the highest point) is not representative of many cases.

Also in the case of the Gaussian bell curve (Figure no.7) the area subtended by the curve is 1 because the sum of the probabilities of all the values gives 1, which is the certainty. The Gauss's theory is also known as the theory of rationality. Indeed, the events that are found on the highest peak of the curve represent those with the highest frequency (and then with the highest probability), while the events with lower probability, closer to the asymptotes, according to this theory are considered negligible, only because of the fact they are rare, regardless of their importance in terms of damage they can produce.

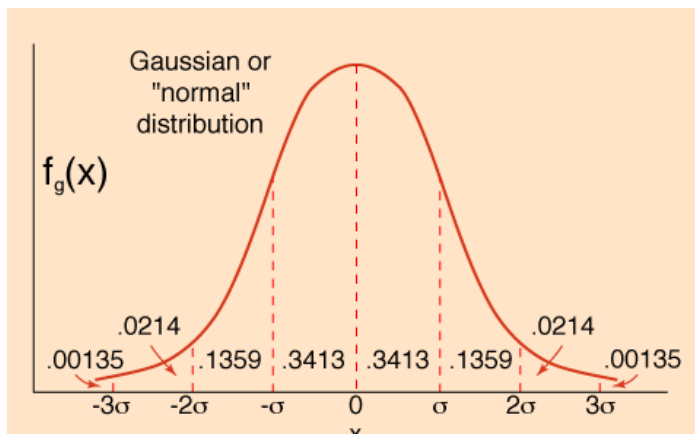


Figure 7. Gaussian Bell – Normal Distribution

### 3.2. Fractals and Scaling in Finance

With fractals the rules are precise and the result is predictable. This contrasts with traditional science which instead includes the irregular aspects of nature and non-similar events such as chaos theory. It is chaos theory a drop of water that expands into the sea, or cardiac fibrillations, or even computer errors and price fluctuations.

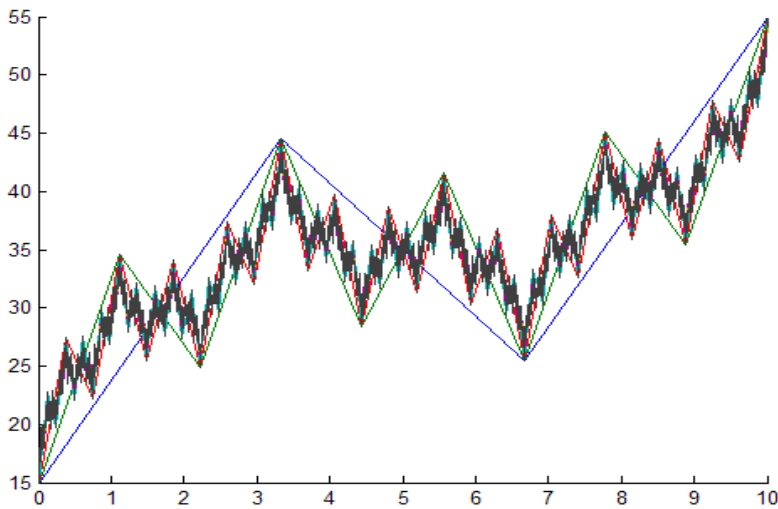


Figure 8. Fractal stock price simulation

Nevertheless, sometimes reality overcomes that the chaos theory exists, in the sense that the unpredictable is realized. This is the collapse of the stock exchange in 1929, or the unfortunate financial events of August 1998, and financial crisis from 2008. According to the standard models, studied by the traditional economy, the sequence of these events was so unlikely as to be impossible. Technically it was called *erratic value*, that is, very far from the normal expected value in the financial world. Yet it happened. This, according to the fractals, means that the traditional economy may have error. Studying of financial markets risk, using fractal theory applicators, it can be offered a new tool to perform better quantitative control and take a better qualitative financial management and investment decisions. The goal is therefore to study the risk, even if Mandelbrot himself admits that nothing can be predicted accurately. It is true that observing the behavior of those who play on the financial market there is something illogical. We observe the stock exchange phenomenon: prices are very variable, movements have an irregular trend (figure no.8). Those who bet on these tendencies to accumulate wealth, they usually put us back because the changes are valued without order: prices increase then without warning, this tendency is interrupted and the opposite trend can even be established (figure no.9).



Figure 9. Fractal Pattern of bitcoin price

On financial markets, the reality overcomes the chaos theory in the sense that the unpredictable is realized as for example the collapse of the stock exchange in 1929 or the unfortunate financial events of August 1998. According to the standard models, that is the models studied by the economy traditional, the sequence of these events was so unlikely as to be impossible. Technically it was called *erratic value*, that is, very far from the normal expected value in the stock world. Yet it happened. This, according to the fractals, means that the traditional economy is in error. Financial markets are risky, everyone knows, but a thorough study of risk, according to fractal theory applicators, can offer a new understanding and one can hope to have quantitative control. The goal is therefore to study the risk, even if Mandelbrot himself admits that nothing can be predicted accurately. It is true that observing the behavior of those who play on the stock market there is something illogical. We observe the stock exchange phenomenon: prices are very variable, movements have an irregular trend. Those who bet on these tendencies to amass wealth, they usually put us back because the changes are valued as without order: prices increase then without warning, this tendency is interrupted and the opposite trend can even be established.

The fractal indicator is based on a simple pricing model that is often found in financial markets. A fractal is a given geometric model that is repeated along all time frames. From this concept was created the fractal indicator. The indicator generates potential turning points on the chart. That shows that the price could move higher and a fractal signal shows the signal that the price might decrease. Bearish fractals are marked by an up arrow and the bullish fractals are shows with an arrow (figure no.10).



Figure 10. Scenarios' tree for financial decision making

The financier Bill Williams has developed the fractal indicator it and can be used in all markets and in all time periods [5]. When there is a low point with two bars or candles a bullish fractal occurs. Moreover when there is a high point a downward fractal occurs. When the arrow is located above or below the center, there is no possibility that a trader can enter the world also if the model is five bars. If someone is exchanged fractal signals, the voice would be the right price of the third bar after the arrow, moreover fractals are frequently used, and sometimes they can be very inaccurate entry points. Fractals can be very inaccurate entry points because they are frequently used. Williams is the one that has

fundament a new indicator, the *Alligator*, among the classical known indicators of the financial stock exchange market. This is an indicator of the combined approach of the direction of travel assessment and filtering market periods of trends absence. The Alligator takes into account the combined analysis of three lines of balance (jaw, teeth and lips). Its interpretation is made as follows: in periods of consolidation, the lines are braided together (sleeping alligator), the more this process lasts, the stronger is this movement of the price (becomes more hungry alligator). Along the trend development, the alligator opens its jaws and follows its prey, the price. As it is more than enough, he squeezes its jaws and falls asleep again [6].

Traders always look for new strategies in their market analysis in order to gain an advantage. A fractal pattern is a model repeated that shows the same configuration throughout the structure, on different scales. Outside, in nature we may see many fractals pattern such as the twigs linked to the tree or snowflakes, these reinforce the properties of being self-similar. No matter how complex is the whole situation, the fractal patterns are recursive since it is built from many repetitions of the same process. If you look at a small section of the pattern, they are self-similar, there is no difference to a much larger section of the pattern, or even the whole.

Fractal patterns can be easily linked with financial markets. Price movement in the market are very complex and appears to be randomly. The financial trader and analyst Bill Williams [5] believes that this complexity is built up from self-similar patterns of trader behavior. That means that the whole structure is not random but follows a model that can be fathom. Williams believes that the structure of the Elliott Wave is actually fractals. The Elliott *Motive Wave* is the first half of the idealized Elliott Wave pattern [7]. It always advances in the direction of the trend of one larger degree and it is subdivided into five smaller waves (Figures no.11). If trades are the results of a behavioral fractal, Williams reasoned, then the aggregate compartment also follows a fractal pattern [7]. That means that the behavioral fractal model gives a way to the traders to potentially profit from the market.



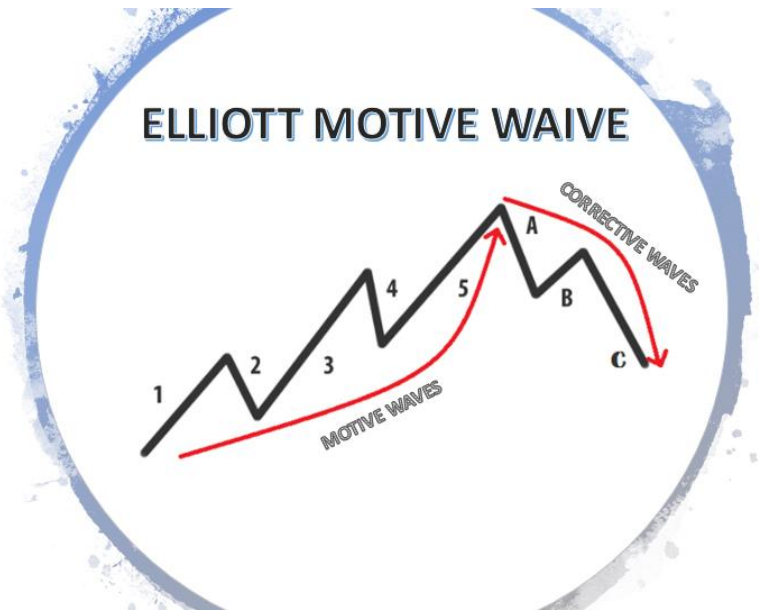


Figure 11. Elliott Motive Waive

#### 4. FINDINGS AND CONCLUSIONS

The Authors tried to reduce the scale of observation and detect the phenomenon by applying the vision of fractal finance. The irregular tendencies of the stock exchange are grouped by size: the large variations arrive in rapid succession followed by sequences of small variations. The behavior of the stock exchange is therefore a fractal structure. In the same way it is possible to proceed with the description of the *bubbles* of the investments, that is the abnormal expansion of a value. The bubbles, however calamitous they may seem, are very common in the general market indices (for example the Dow Jones) and in the individual activities [8]. Despite this, traditional economic models consider the bubbles of aberrations, of irrational deviations of the norm, caused for example by an avid speculator. Why it is not considered as a combined result of so many discontinuities? Or again, why does traditional finance presuppose that the financial system is a linear and continuous machine even if it admits the existence of bubbles?

An example can help: based on the standard model of finance (the price bell curve) the probability of ruin is equal to 1 in ten billion billions, that is more likely to be hit by a meteorite that will go bankrupt in a financial market. But if prices have wild variations (it happened for the price of cotton but also with oil) the probability of ruin increases dramatically.

The most famous and most general pricing model of the options was developed at the beginning of the seventies by Fisher Black and Myron Scholes [9]. Originally this model was developed to price European-style financial options (type of option that cannot be reimbursed before their maturity). Starting from that first early version it has contributed and influenced all subsequent pricing models. An important contribution to the development of the model of Black and Scholes undoubtedly goes to Merton who, on the basis of the 1973 version, made changes and improvements [10]. According to their

binomial model, the basic hypothesis is the possibility of creating a portfolio equivalent to the option, consisting partly of units of the underlying and partly of risk-free bonds. The main difference with respect to the binomial model is that in this case the hypothesis foresees that the yields are distributed among infinite states of nature according to a normal Gaussian distribution. The Black and Scholes model represents the limit in the continuum of the binomial model, which is discrete (figure no.12).

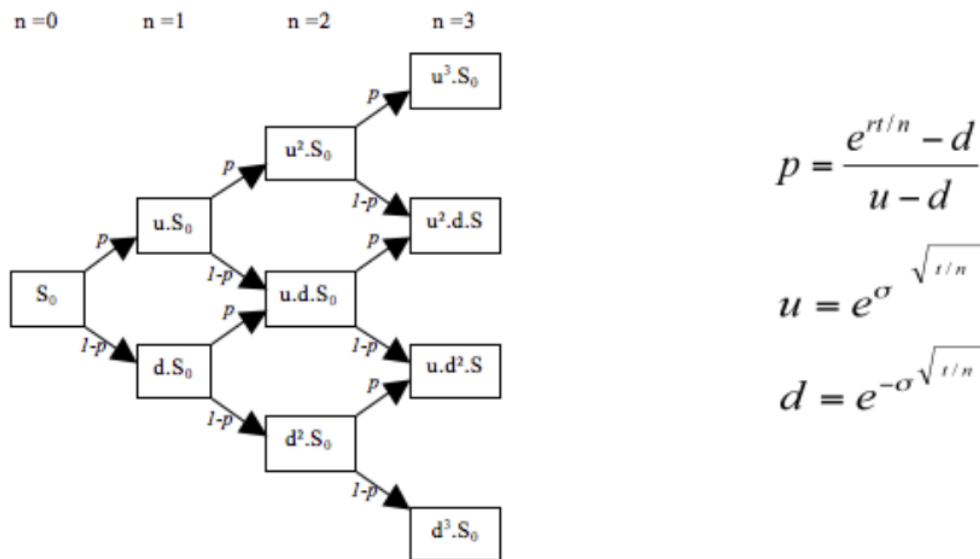


Figure 12. Binomial option pricing model. Decision tree approximation.

The intention and suggestion of the Authors is to apply an innovation to the Black and Scholes method, basing the analysis of the pricing of the real options no longer on the normal distribution, but on a fractal approach. According to authors' opinion, this method, despite all evidence, definitively can be the one which represents all possible states of nature and all scenarios, without neglecting any event, not even those that are unlikely to be happened.

Since decision-making strategies should be based on the most detailed analysis possible of all scenarios, the normal distribution therefore appears to be a limit rather than a foundation, as it excludes unlikely events. Moreover, it has been shown that the most improbable events are the most dangerous, or the one that represent the most important opportunities, therefore they are not negligible.

Future research will be able to focus on quantitative and econometric analysis, with the support of computing power and computer design, and able to discover and test complexity and the perfectly realistic algorithms.

In essence, with the implementation of the model suggested in this paper, it will be possible to replace the probability percentages assigned (currently by normal distribution) to each branch of the Black and Scholes construction, with more detail through the fractal approach.

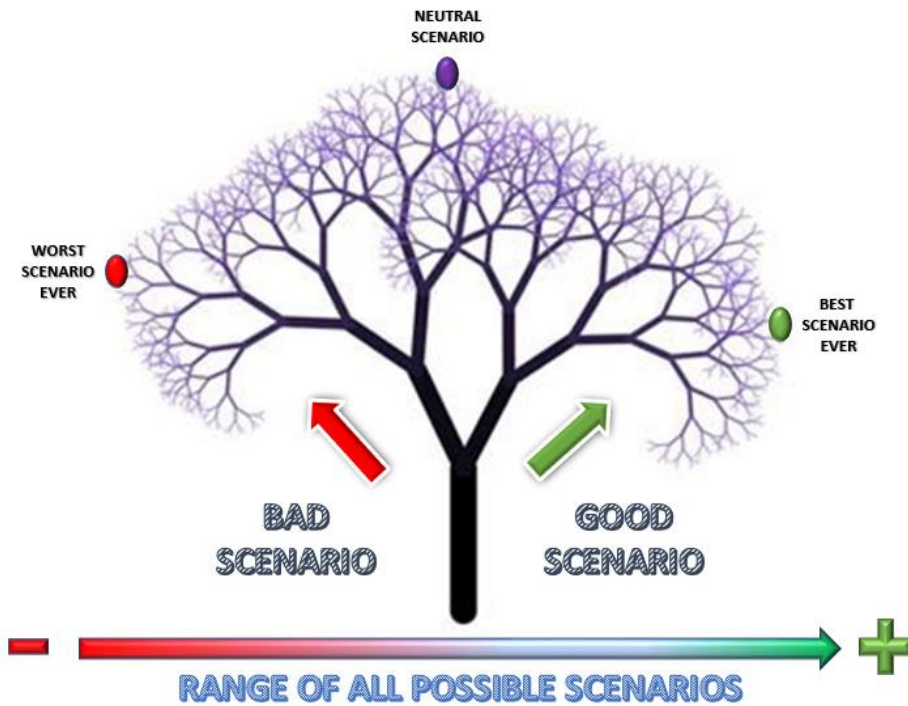


Figure 13: Scenarios' tree for financial decision making

Financial performance of foreign exchange markets or performance of foreign investments [11] is a constant issue of the present normal or turmoil economic, however it is insufficiently explored, without taking into consideration all aspects, like fractals patterns. The analysis of the economic and financial performances is a frequently debated matter in the economic media from the last decades, covering a very large spectrum, comprising various meanings and tendencies which will continuously capture the interest of the economists, the accountants and the IT specialists [12]. However, the present research comes to emphasize that fractals patterns can predict a certain economic trend for each type of transaction or economic behaviour, under the circumstances of the action of the same type of external factors [13].

A multidimensional analysis of financial data provides a clear picture of decision makers at company level or at macroeconomic level. Economic data are very useful and they can be classified according to different decision maker criteria and taking into consideration accordingly. The research showed and natural life confirm that patterns exist in economic life too. These patterns may help, even may become an important tool for financial decision makers. However, for a better performance of economic and financial forecasts, predictions and the impact of decisions throughout the economy, we argue that fractals pattern are more than helpful be taken into account, fractals can let financial experts to be aware or prepared for periods of cyclical recession and take better and less risky business decisions.

## REFERENCES

- [1] Mandelbrot B.B., 1997. A case against the lognormal distribution. Book - Fractals and Scaling in Finance. *Springer*.
- [2] Taleb, N.N., 2007. The black swan: The impact of the highly improbable (Vol. 2). *Random house*.
- [3] Mandelbrot B.B., 2005. The inescapable need for fractal tools in finance. *Annals of Finance*.
- [4] Barbosa P.M.R, 2011, Chaos Theory and Mathematics, the relation between formal science and natural science, *Underdetermination of Science*.
- [5] Williams B., 1998. New Trading Dimensions: How to profit From Chaos in Stocks, Bonds, and Commodities, *John Wiley and Sons*.
- [6] Kagitci, M., Nichita, M.E., Vulpoi, M., & Paunescu, M., 2012. A survey of evolution of profitability indicators of pharmacy entities listed on Bucharest Stock Stock Exchange before and during the global financial crisis. *Proceedings of the 7<sup>th</sup> International Conference Accounting and Management Information System AMIS 2012*.
- [7] Frost, A.J. and Prechter, R.R., 2005. Elliott wave principle: key to market behavior. *Elliott Wave International*.
- [8] Shiller, R.J., 2000. Measuring bubble expectations and investor confidence. *The Journal of Psychology and Financial Markets*, 1(1), pp.49-60.
- [9] Black, F., and Scholes, M., 1973. The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*.
- [10] Merton, R., 1973. Theory of Rational Option Pricing, *Bell Journal of Economics and Management Science*.
- [11] Moșteanu, N.R., 2017. Currency pegged to a foreign currency- GCC and Europe models, *The Business and Management Review*, Vol.9 No.1.
- [12] Boldeanu, D.M., & Gheorghe, M., 2012. Multidimensional analysis of firm's financial performance through the influence factors in pharmaceutical sector. *Proceedings of the 7<sup>th</sup> International Conference Accounting and Management Information System AMIS 2012*
- [13] Mosteanu, N.R, Faccia A., Torrebruno, G. and Torrebruno, F., 2019. The newest intelligent financial decision tool: fractals. A smart approach to assess the risk. *Proceedings of 8<sup>th</sup> International Conference on Business and Economic Development (ICBED)*. 8-9 April, New York, USA